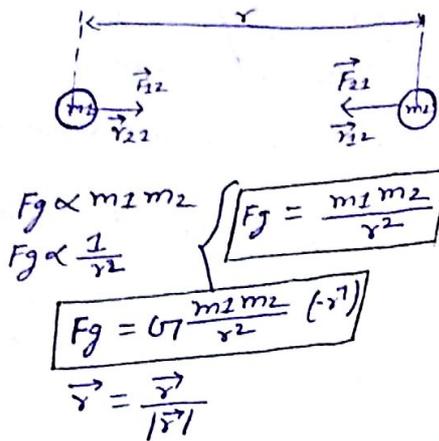


# Gravitation



$$F_g = G \frac{m_1 m_2}{r^3} (-\vec{r})$$

$$\vec{F}_{12} = \frac{G m_1 m_2}{r^3} (\vec{r}_{12})$$

$$\vec{F}_{21} = \frac{G m_1 m_2}{r^3} (\vec{r}_{21}) = \frac{G m_1 m_2}{r^3} (-\vec{r}_{12})$$

$$\vec{F}_{21} + \vec{F}_{12} = 0$$

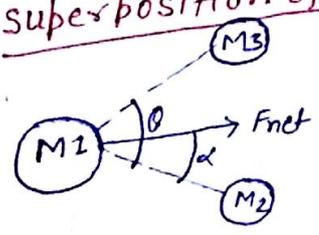
$$\vec{F}_{12} = -\vec{r}_{21}$$

$$\vec{F}_{21} = -\vec{F}_{12}$$

**\*  $G$  = universal gravitational const.**  
 $= 6.67 \times 10^{-11} \text{ Nm}^2/\text{kg}^2 \text{ (S.I.)}$   
 $= 6.67 \times 10^{-8} \text{ (cgs)}$   
**ALPMT**  
**\* Dimension  $\Rightarrow M^{-1} L^3 T^{-2}$**

- NOTE**
- AIIMS 2015** \* It is only attractive nature force & its direction is always towards mass.
  - AI\*** \* It is **medium independent force**.
  - AI\*** \* Gravitational force on a particle inside spherical shell is zero. However (unlike a metallic shell which shields electric forces & magnetic forces) the shell does not shield other bodies outside, it from external gravitational forces on particle inside. Gravitational shielding is not possible!!
  - AI\*** \* Gravitational force field is conservative force field. Work done is independent from path.
  - AI\*** \* In case of gravitational force field net force on system is zero that's why linear momentum conserved.
  - AI\*** \* Line of Action of gravitational force is passed from fixed point & it obey inverse square law that's why it is called **central force**.
  - \*\* !! \*** \* Gravitational force b/w two mass remain unchange with time of another mass.

## # Superposition of force $\rightarrow$



$$F_{net} = F_1^2 + F_2^2 + 2F_1 F_2 \cos \theta$$

$$\alpha = \tan^{-1} \left( \frac{F \sin \theta}{F_1 + F_2 \cos \theta} \right)$$

**\* Net Gravitational force is vector Addition of Individual force.**  

$$\vec{F}_{net} = \vec{F}_1 + \vec{F}_2 + \dots + \vec{F}_N = \sum_{i=1}^N \vec{F}_i$$

- NOTE**
- \*\*\*** \* In a Regular polygon If similar mass is placed at the vertex net force at central mass **zero**.
  - \* If  $(n-1)$  similar mass placed at the vertex of regular polygon net force on central mass equal to  $[F]$ .

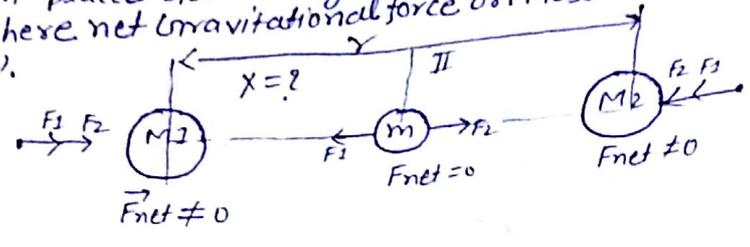
# similar mass 'm' placed along x-axis at  $x=0, x=a, x=2a, x=4a$  & up to  $\infty$  calculate net gravitational force on mass 'm' at  $x=0$ .  $\rightarrow F_{net} = \frac{4}{3} \frac{GM^2}{a^2}$

# Mass 'M' divide into 2-parts & placed at fixed distance calculate maximum value of force b/w divide mass. If distance b/w mass is  $x \rightarrow m = \frac{M}{2} \left[ \frac{GM^2}{4x^2} \right]$

# Two sphere of Radius 'R' mass 'm' placed in contact calculate net gravitational force on one sphere  $\rightarrow \sqrt{3GM^2}$

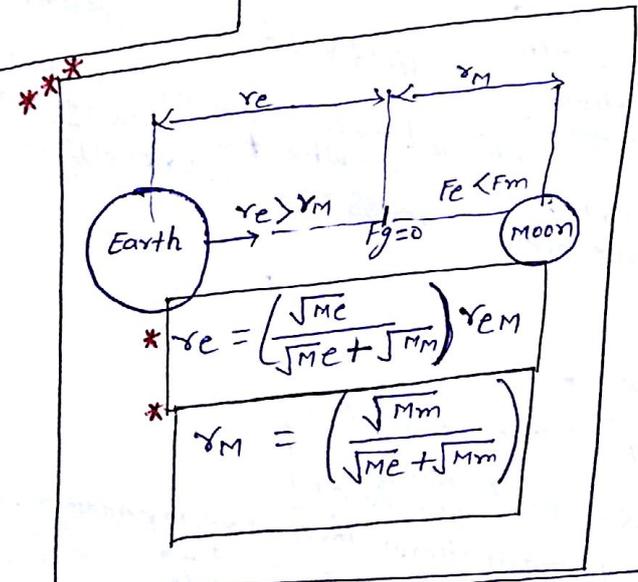
# Two similar Mass 'm' Rotate in a circular path of Radius 'R' due to mutual gravitational attractive force. calculate velocity of each mass.  $\rightarrow \sqrt{\frac{2GM}{2R}}$

# Mass 'm' placed b/w joining line of  $M_1$  &  $M_2$  calculate distance from mass  $M_1$  where net gravitational force on mass  $m$  is zero distance b/w  $m_1$  &  $m_2$  is 'x'.



$F_1 - F_2 = 0$   
 $F_1 = F_2$   
 $\frac{GM_1 m}{x^2} = \frac{GM_2 m}{(y-x)^2}$   
 $\sqrt{\frac{M_1}{x}} = \sqrt{\frac{M_2}{y-x}}$   
 $* X = \frac{\sqrt{M_1 y}}{\sqrt{M_1} + \sqrt{M_2}}$

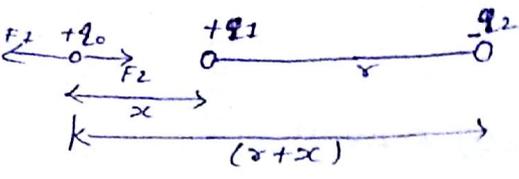
$* y - x = \frac{\sqrt{M_2 y}}{\sqrt{M_1} + \sqrt{M_2}}$



# CASE-I (When  $q_1, q_2$  are like charge)  $\rightarrow$

$* x = \frac{\sqrt{q_1} y}{\sqrt{q_1} + \sqrt{q_2}} \rightarrow$  From  $q_1$   
 $y - x = \frac{\sqrt{q_2} y}{\sqrt{q_1} + \sqrt{q_2}} \rightarrow$  From  $q_2$

# case-II (When  $g_1, g_2$  are unlike)  $\rightarrow$



$$x = \frac{\sqrt{g_1} y}{\sqrt{g_2} - \sqrt{g_1}} \rightarrow \text{From } q_1$$

$$x + x = \frac{\sqrt{g_2} y}{\sqrt{g_2} - \sqrt{g_1}} \rightarrow \text{From } q_2$$

\*\*# Rod of mass 'M' length 'L' placed in a horizontal plane & mass 'm' is placed at distance 'x' on the axis of rod calculate gravitational force on mass m  $\rightarrow$

$$F = \frac{GMm}{x(L+x)}$$

# Gravitational Accn.

Accn. of particle in presence of gravitational force field.

$$a = \frac{F_g}{m}$$

$$g = \frac{GM}{R^2}$$

$$g = \frac{GM}{R^3} = \frac{4}{3} \pi G R \rho$$

M = Mass of planet  
R = Radius of planet  
 $\rho$  = Avg. density of planet.

\*\*\* NOTE  $\rightarrow$

Gravitational Accn. of planet independent from mass of particle  
It only depend on mass of planet, radius of planet & density of planet.

$$g_{\text{earth}} = g = 9.8 \text{ m/sec}^2 = 980 \text{ cm/sec}^2 = 32 \text{ F/sec}^2$$

$$g_m = \frac{g_e}{6}$$

$$g_{\text{Jupiter}} = 1.4 g_e$$

# Effecting factor of 'g'

[A] Effect of Height  $\rightarrow$

$$g_h = \frac{g_s}{(1+h/R)^2}$$

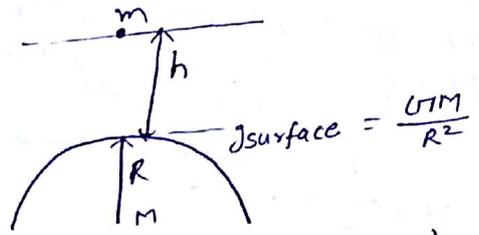
\*  $h \ll R$  or 'h' is  $\leq 5\%$  of R

$$h \leq \frac{5}{100} \times 6400$$

$$h \leq 320 \text{ km}$$

$$g_h = g_s \left(1 + \frac{h}{R}\right)^{-2}$$

$$h \ll R = \frac{h}{R} \ll 1$$



$$(1+x)^n = 1 + nx \quad (x \ll 1)$$

$$g_h = g_s \left(1 - \frac{2h}{R}\right)$$

Exemplar

\*# |a|  $\rightarrow$  'h' is in Range of 'R'

$$g_h = g_s / (1+h/R)^2$$

|b|  $\rightarrow h \leq 320 \text{ km} (\leq 5\% \text{ of } R)$

$$g_h = g \left(1 - \frac{2h}{R}\right)$$

\*  $h \uparrow \Rightarrow g_h \downarrow$

$$* 32 \uparrow \rightarrow 1\% \downarrow$$

$$* 64 \downarrow \rightarrow 1\% \uparrow$$

\* % change in gravitational Acc<sup>n</sup> at height above from earth surface -

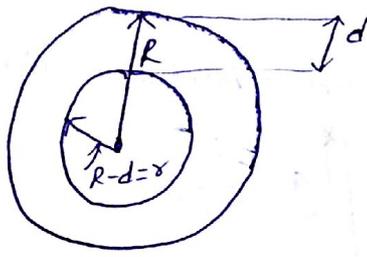
- i)  $h = 32 \text{ km} \rightarrow -1\%$
- ii)  $h = R/2 \rightarrow -55.55\%$
- iii)  $h = 1600 \text{ km} \rightarrow -36\%$

\* Height from earth surface where gravitational Acc<sup>n</sup> remain

- i) 19% of surface  $\rightarrow h = R \left( \frac{10}{\sqrt{10}} - 1 \right)$
- ii) 64% of surface  $\rightarrow h = \frac{2R}{8} = R/4$
- iii) 99% of surface  $\rightarrow 32 \text{ km}$

[B]  $\rightarrow$  Effect of Depth  $\rightarrow$

$$g_d = g_s \left( 1 - \frac{d}{R} \right)$$



- \*  $d \uparrow \Rightarrow g_d \downarrow$
- $d =$  depth from surface.
- \*  $d = R$  (at centre of earth)

$$g_d = g \left( 1 - \frac{R}{R} \right) = 0$$

$$g_d = \frac{4}{3} \pi G \rho R - d$$

↑  
depth from surface.

$$\rho = \frac{4}{3} \pi G \rho r \propto r$$

↘ distance from of centre of earth.

$$g \propto r$$

Imp \* Relation b/w height & depth from earth surface where change in gravitation Acc<sup>n</sup> same ( $h \text{ is } \ll R$ )  $\rightarrow d = 2h$

(उदाहरण 1 km down earth surface पर गुरुत्वाकर्षण 'g' का मान होता है 9.8 m/s<sup>2</sup>।  
 If g is value at earth surface at 12 km down (उदाहरण)  $\rightarrow$   $g$  का मान होगा 9.8 m/s<sup>2</sup>।)

\* If 'h' in range of 'R' compare height & depth for same cond<sup>n</sup>.

$$g_h = g \left( \frac{g}{(1 + \frac{h}{R})^2} \right) = g \left( 1 - \frac{d}{R} \right)$$

[C]  $\rightarrow$  Effect of Rotation  $\rightarrow$

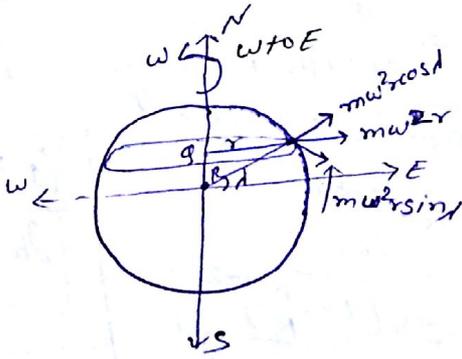
$$W_{\text{eff}} = mg - \omega^2 r \cos \lambda$$

$$W_{\text{eff}} = mg - m\omega^2 R \cos \lambda$$

↓  
 $m g_{\text{eff}}$

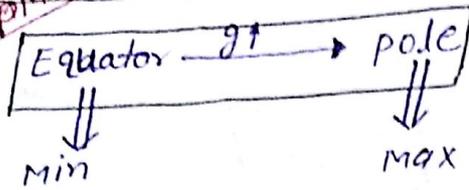
$$g_{\text{eff}} = g - \omega^2 R \cos \lambda$$

- $\lambda =$  latitude angle
- $R =$  Radius of earth
- $\omega =$  Angular speed of earth
- \*  $\omega \uparrow \Rightarrow \cos \lambda \downarrow \Rightarrow g_{\text{eff}} \uparrow$



$$W_{\text{earth}} = \frac{2\pi}{T_{\text{earth}}} = \frac{2\pi}{24 \times 60 \times 60} \text{ RIS}$$

#xxx  
UPCPMT



\*  $\lambda = 0^\circ$  (at equator)

$$g_e = g - \omega^2 R \neq g_{min}$$

\*  $\lambda = 90^\circ$  (at pole)

$$g_p = g = g_{max}$$

Eg  $\rightarrow \omega = m\gamma = \text{same}$

$$m \propto \frac{1}{g}$$

$$g_{eq} < g_p$$

$$m_e > m_p$$

UPCPMT

(Means 1 kg sugar pole pe aur 1 kg chini equator pe Jada)

# Gravity free condition  $\rightarrow g' = g - \omega^2 R \cos^2 \lambda$

|a|  $\rightarrow \lambda = 0^\circ$  (equator line)  $\Rightarrow g_e = g - \omega^2 R$

|b|  $\rightarrow \lambda = 90^\circ$  (poles)  $\Rightarrow g_p = g \propto \omega^0$

$$g_p - g_e = \omega^2 R = 0.03 \text{ m/sec}^2$$

$\downarrow$                        $\downarrow$   
 max                      min

|c|  $\rightarrow$  Gravity free condition  $g' = 0 = g - \omega^2 R \cos^2 \lambda$

$$[\omega \uparrow \Rightarrow g' \downarrow]$$

$$\omega = \frac{1}{\cos \lambda} \sqrt{g/R}$$

$$T = 84.6 (\cos \lambda) \text{ min} \Rightarrow 1.414 (\cos \lambda) \text{ hr}$$

\* At equator  $\lambda = 0 \Rightarrow \cos 0 = 1 \Rightarrow T \Rightarrow 84.6 \text{ min} \Rightarrow 1.414 \text{ hr}$

$$\omega = \frac{2\pi}{T} \Rightarrow \omega' = 17$$

\*\*

NOTE  $\rightarrow$  \* If angular speed of earth become 17 times of  $\omega$  value gravity free condition is  $\oplus$  on equator line.  
 \* For gravity free condition  $\omega_{min}$  at equator.  
 ! \* At pole gravity free condition is not possible.

[D]  $\rightarrow$  Effect of shape  $\rightarrow$

$$r_e = r_p + 21 \text{ km}$$

$$r_e - r_p = 21 \text{ km}$$

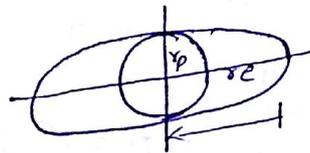
$$* r_e > r_p$$

$$g_e < g_p$$

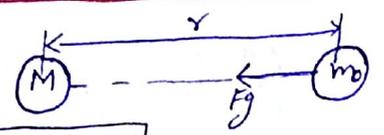
$$* g_e - g_p = 0.02 \text{ m/sec}^2 \rightarrow \text{Due to shape}$$

$$g_p - g_e = 0.03 \text{ m/sec}^2 \rightarrow \text{Due to Rotation}$$

$$* g_p - g_e = 0.05 \text{ m/sec}^2$$



# Gravitational field Intensity ( $I_g$ )  $\rightarrow$  Force per unit in gravitational field.



$$I_g = \frac{F_g}{m_0} = \frac{G M m_0}{r^2} \cdot \frac{1}{m_0}$$

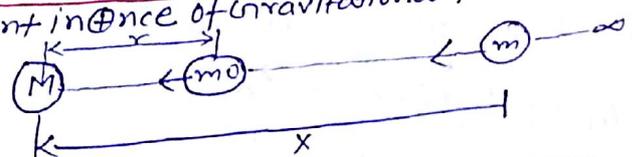
$$I_g = \frac{G M}{r^2} \rightarrow \text{vector quantity.}$$

$$I_g = \frac{G M}{r^2} (-\hat{r}) = -\frac{G M}{r^3} \vec{r}$$

- \* Direction is in the direction of force.
- \* unit  $\rightarrow \frac{N}{kg} = \frac{m}{sec^2}$

NOTE  $\rightarrow I_g$  depend on mass of object & distance from mass it is independent from the mass of testing object.

# Gravitational potential ( $V_g$ )  $\rightarrow$  Work done to bring a mass from  $\infty$  to specific point in  $\oplus$ nce of gravitational field.



$$(V_g)_p = \frac{W_{oop}}{m_0}$$

$$W_{oop} = -\frac{G M m_0}{r}$$

$$(V_g)_p = \frac{W_{oop}}{m_0} = -\frac{G M}{r}$$

$\rightarrow$  show attractive force field.

- NOTE  $\rightarrow V_g$  is a scalar quantity.
- \* unit  $\rightarrow \frac{J}{kg} \neq \text{volt}$

\*\*  
\* Relation b/w  $I_g$  &  $V_g$

$$I_g = -\frac{\Delta V_g}{\Delta r}$$

$$-\Delta V_g = -\int I_g \Delta r$$

# Similar mass is placed on x-axis at  $x=0, x=a, x=2a, x=4a, x=8a$  upto  $\infty$  then

- i)  $I_g \rightarrow -\frac{4}{3} \frac{G M}{a^2} \hat{r}$
- ii)  $V_g \rightarrow -\frac{2 G M}{a}$

\*\*

$$I_g = -\frac{\Delta V_g}{\Delta r}$$

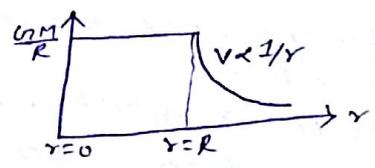
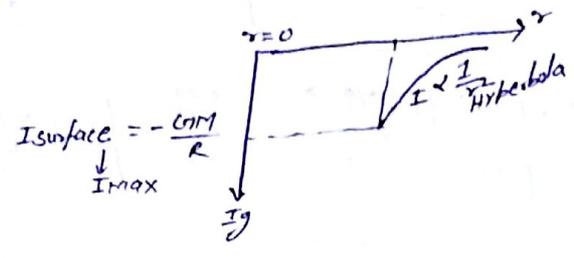
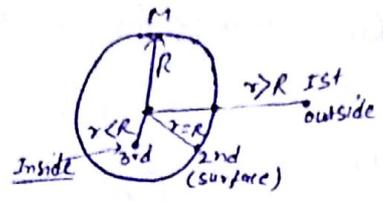
1a)  $V_g = f(r) \Rightarrow I_g = -\frac{d}{dr} [f(r)]$

1b)  $I_g = f(r) \Rightarrow \int dV_g = -\int I_g dr$

\*\*\*  
# Gravitational field Intensity & potential for continuous mass distribution obred.

[A] → Hollow sphere →

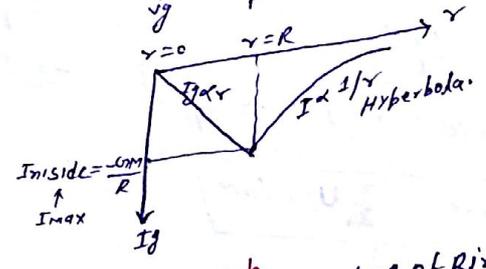
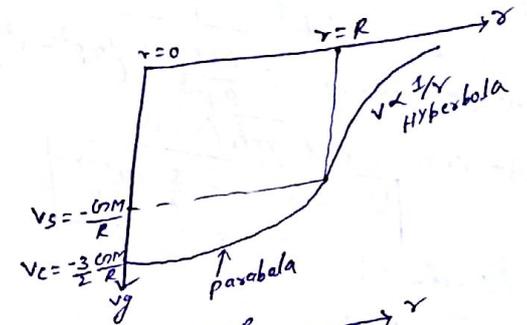
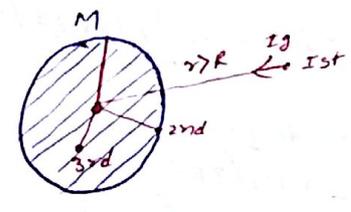
- ii)  $r > R$  (outside) ⇒  $I_g = -\frac{GM}{r^2}$   
 $V_g = -\frac{GM}{r}$
- iii)  $r = R$  (surface) ⇒  $I_g = -\frac{GM}{R^2}$   
 $V_g = -\frac{GM}{R}$
- iiii)  $r < R$  (inside)  
 $I_g = 0$      $V_g = -\frac{GM}{R}$



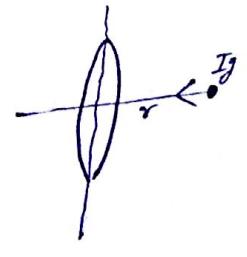
or, Earth planet

[B] → Solid sphere →

- ii)  $r > R$  (outside)  $I_g = -\frac{GM}{r^2}$   
 $V_g = -\frac{GM}{r}$
- iii)  $r = R$  (surface)  $I_g = -\frac{GM}{R^2}$   
 $V_g = -\frac{GM}{R}$
- iiii)  $r < R$  (inside)  
 $I_g = -\frac{GM r}{R^3}$   
 $V_g = -\frac{GM(3R^2 - r^2)}{2R^3}$
- lv) At centre ( $r=0$ )  
 $I_g = 0$   
 $V_{centre} = -\frac{3}{2} \frac{GM}{R}$   
 $V_{centre} = \frac{3}{2} V_{surface}$



[C] → Ring →



\*  $I_g = -\frac{GM r}{(R^2 + r^2)^{3/2}}$   
\*  $V_g = -\frac{GM}{(R^2 + r^2)^{1/2}}$

Imp \* At centre of Ring ( $r=0$ )

$I_g = 0$   
 $V_g = -\frac{GM}{R}$

# Gravitational pot. Energy (G.P.E) → In presence of gravitational force field Work done for bringing a mass from  $\infty$  to sp. point.

\* Work done against force field stored in system in form of P.E.

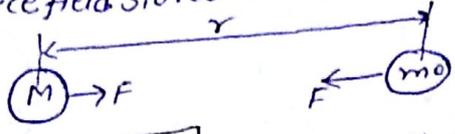
$$v F = - \frac{du}{dr}$$

$$- \int du = - \int F \cdot dr$$

$$v F = - \frac{GMm_0}{r^2}$$

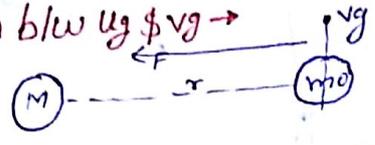
$$U = - \frac{GMm}{r}$$

\* At  $r = \infty, U_0 = 0$



\*\*

# Relation b/w  $U_g$  &  $v_g \rightarrow$



$$U = v_g(m_0)$$

- NOTE →
- i) → G.P.E of any one mass in 'n' mass system is equal to scalar addition of (n-1) pair.
  - ii) → G.P.E of 'n' mass system is scalar addition of  $\frac{n(n-1)}{2}$  pair.
  - iii) → G.P.E is always  $\ominus$ ve bcoz it is attractive nature force.

\*\*

# G.P.E of Earth/planet/solid sphere →

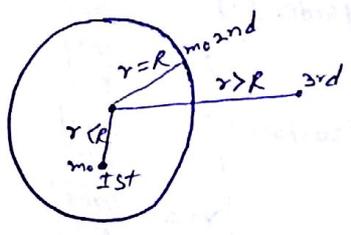
ii) →  $r < R$  (Inside)

$$v_g = - \frac{GM(3R^2 - r^2)}{2R^3}$$

$$* U_{\text{inside}} = m v_g = - \frac{GMm}{2R^3} (3R^2 - r^2)$$

\* At centre of Earth ( $r=0$ )

$$U_{\text{centre}} = - \frac{3}{2} \frac{GMm}{R}$$

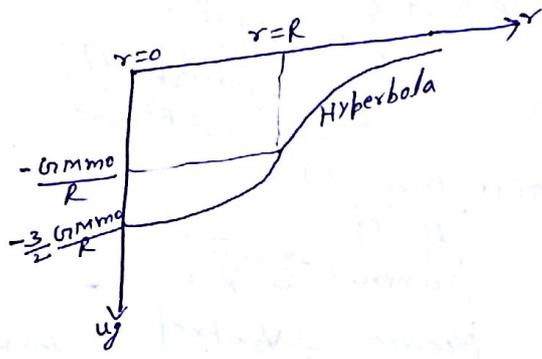


iii) →  $R = r$  (surface)

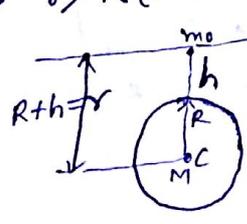
$$v_g = - \frac{GM}{R}$$

$$* U_{\text{surface}} = m v_g = - \frac{GMm_0}{R}$$

$$U_{\text{centre}} = \frac{3}{2} U_{\text{surface}}$$



iiii) →  $r > R$  (outside)

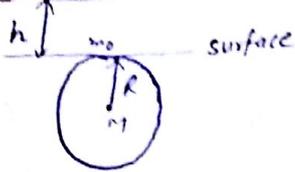


$$v_g = - \frac{GM}{r} = - \frac{GM}{R+h}$$

$$* U_g = m v_g = - \frac{GMm}{r} \propto 1/r$$

$$U_g = - \frac{GMm_0}{R+h}$$

Work done  $\rightarrow$



$$W_{ext} + W_{nc} = \Delta K.E + \Delta U$$

$$g_{surface} = \frac{GM}{R^2}$$

$$W_{ext} = \frac{mgh}{1 + \frac{h}{R}}$$

$$* h \ll R \Rightarrow \frac{h}{R} \ll 1 \quad * W = mgh$$

neglect

# Max height attained by particle  $\rightarrow$

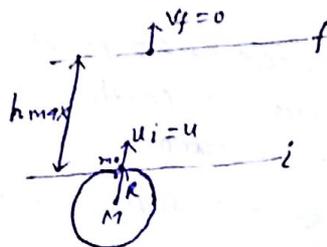
$$W_{nc} = 0 = W_{ext}$$

$$0 + 0 = \Delta K.E + \Delta U$$

$$KE_i + U_i = KE_f + U_f$$

$$* v = \sqrt{\frac{2gh}{1 + h/R}}$$

$$* h_{max} = \frac{v^2 R}{2g - v^2} \quad (v < v_{escape})$$



# Escape velocity ( $v_{escape}$ )  $\rightarrow$  Min Required velocity of particle to remove from gravitational force field.

[A]  $\rightarrow$  At surface of planet  $\rightarrow$

$$v_{min} = \sqrt{\frac{2GM}{R} + v_{\infty}^2}$$

zero

$$* v_{escape} = \sqrt{\frac{2GM}{R}}$$

$$g_{surface} = g = \frac{GM}{R^2} \rightarrow \text{Accn due to gravity.}$$

$$* (v_{escape})_{surface} = \sqrt{\frac{2GM}{R}} = \sqrt{2Rg} = \sqrt{\frac{8}{3} \pi G R^2 \rho}$$

NOTE  $\rightarrow$  \* Escape velocity depend on mass, radius & Avg. density of planet but it is independent from mass of particle, protection angle & protection direction & Rotation of Earth.

iii  $\rightarrow$  At 'h' height above from earth surface / planet surface

$$v_{escape} = \sqrt{\frac{2GM}{r}} = \sqrt{\frac{2GM}{R+h}}$$

i)  $\rightarrow v_{surface} < v_{escape}$

$$h_{max} = \frac{v^2 R}{2g - v^2}$$

velo. at max ht = 0

ii)  $\rightarrow v_{surface} = v_{escape} \Rightarrow h_{max} = \infty$  (particle escape from planet)  
 $v_{\infty} = 0$

iii)  $\rightarrow v_{surface} > v_{escape}$

$$h_{max} = \infty$$

$$v_{\infty} = \sqrt{v_{surface}^2 - \frac{2GM}{R}}$$

$$v_{\infty} = \sqrt{v_s^2 - v_{escape}^2}$$

$$* \begin{matrix} V_{earth} = 11.2 \text{ km/sec} \\ V_{moon} = 3.4 \text{ km/sec} \end{matrix} \quad \left| \quad V_{orbit} = \sqrt{\frac{GM}{r}}$$

# Max height attain by particle & its velocity at max height. If velocity of particle at surface of earth -

(i)  $V_s = \sqrt{\frac{GM}{R}} = R$  \* at  $v = h_{max} = 0$

(ii)  $V_s = \sqrt{\frac{3GM}{2R}} < V_{escape} \Rightarrow v = 3R$

(iii)  $V_s = \sqrt{\frac{3GM}{R}} > V_{escape} \Rightarrow h_{max} = \infty$  \*  $V_{\infty} = \sqrt{\frac{GM}{R}}$

# velocity of particle at  $R/2$  height above from earth surface is 'f' times of  $V_{escape}$  at surface of earth. Find out value of 'f'. If particle just escape from gravitational force field of planet  $\rightarrow f = \sqrt{2/3}$

# particle of mass 'm' is drop along the diametric tunnel of earth. Find out velocity of particle -

(a)  $\rightarrow$  At centre of earth  $\rightarrow V_c = \sqrt{\frac{GM}{R}} = \frac{V_{escape}}{\sqrt{2}}$

(b)  $\rightarrow$  At opposite end of tunnel  $\rightarrow V_b = 0$

# particle of mass 'm' drop along the diametric tunnel of earth. then time period of particle  $\rightarrow T = \frac{2\pi}{\omega} = 2\pi \sqrt{\frac{R}{g}} = 84.6 \text{ min} = 1.414 \text{ hr}$

## ORBITAL MOTION

(1)  $\rightarrow$  orbital velocity  $\rightarrow$

$$V_o = \sqrt{\frac{GM}{r}} = \sqrt{\frac{GM}{R+h}}$$

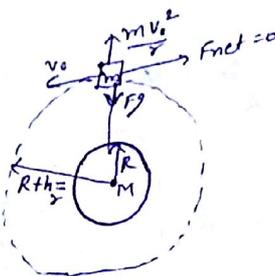
\*  $h \ll R \Rightarrow \left(\frac{h}{R}\right) \ll 1$  (near above the earth surface)  $\leftarrow$  neglect

$$V_o = \sqrt{\frac{GM}{R}} = \frac{V_{escape}}{\sqrt{2}}$$

\* satellite near about earth surface

$$V_{orbital} = \frac{V_{escape}}{\sqrt{2}} = 0.707 \times 11.2 \text{ km/sec}$$

$$V_o = 7.9 \text{ km/sec} = 8 \text{ km/sec.}$$



(2)  $\rightarrow$  Time period  $\rightarrow$

$$T = \frac{2\pi}{\omega} \propto r^{3/2}$$

$$T = 2\pi \sqrt{\frac{R}{g}} \left(1 + \frac{h}{R}\right)^{3/2}$$

\* Time period of satellite of Earth.

$$T = 84.6 \left(1 + \frac{h}{R}\right)^{3/2} \text{ min} = 1.414 \left(1 + \frac{h}{R}\right)^{3/2} \text{ hr.}$$

(3)  $\rightarrow$  Energy of satellite in a stable circular orbit  $\rightarrow$

[A]  $\rightarrow$  kinetic energy  $\rightarrow$

$$K.E = \frac{2Mm}{2r} = \frac{GMm}{2(R+h)}$$

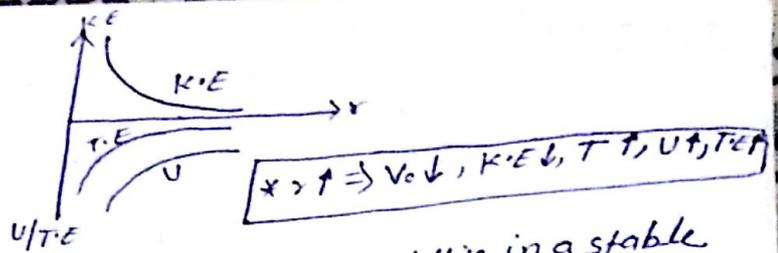
[B]  $\rightarrow$  Potential energy  $\rightarrow$

$$U = -\frac{GMm}{r}$$

[C] → Total Energy / M. Energy →

$$T.E = -\frac{GMm}{2r}$$

$$T.E = -K.E = \frac{P.E}{2}$$



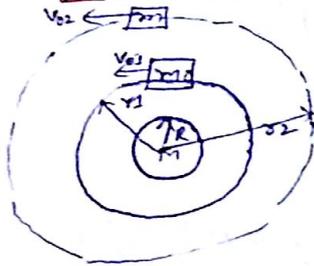
[D] → Binding Energy (B.E) → Required energy to bound the satellite in a stable orbit or, Require min energy to remove the satellite from stable orbit.  
\* It is also called Escape Energy.

$$* K.E_R = \frac{GMm}{2r} \text{ (Require energy to escape the satellite)}$$

$$B.E = \frac{GMm}{2r} = \text{Escape energy} = K.E = -T.E$$

- NOTE** → \* If K.E of particle in a stable orbit become double or, ↑ 100% satellite escape from planet.  
\* If T.E of satellite in a stable orbit become zero. It will be escape from gravitational force field of planet.  
\* If orbital velocity become  $\sqrt{2}$  times or, ↑ 41.4% satellite escape from G. Force field of planet.

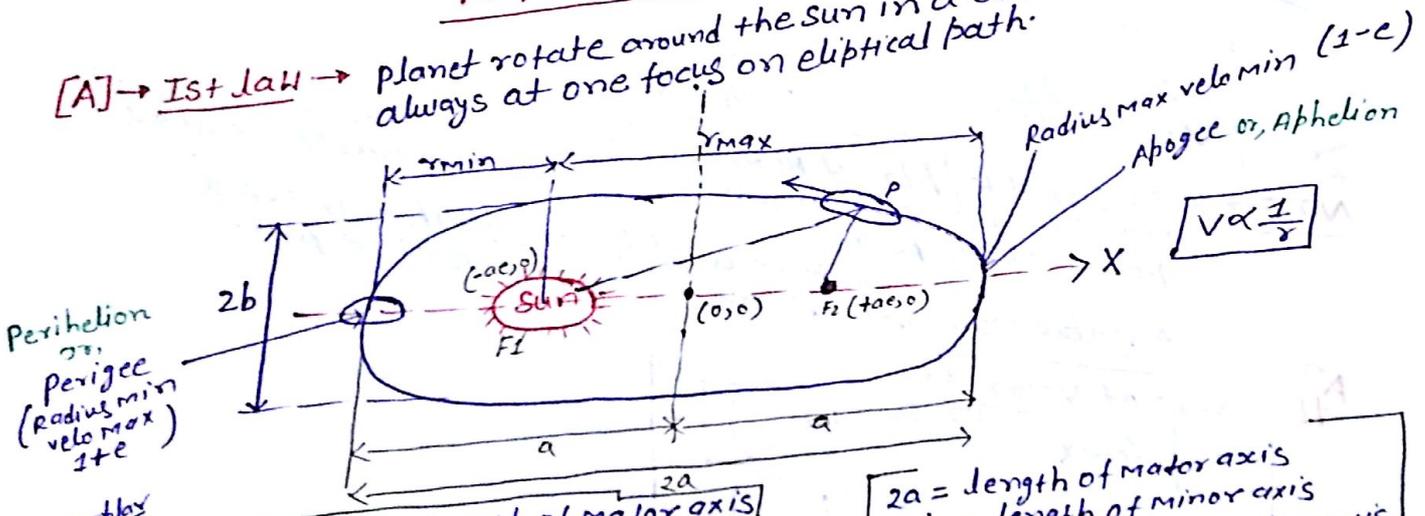
[E] → Work done to change the orbit of satellite



$$* W = \frac{GMm}{2} \left( \frac{1}{r_1} - \frac{1}{r_2} \right)$$

### 'Kepler's Law'

[A] → 1st Law → planet rotate around the sun in a elliptical path & sun ⊕ nt always at one focus on elliptical path.



Exemplar

$$PF_1 + PF_2 = \text{const} = \text{length of major axis}$$

$$r_{\min} + r_{\max} = 2a$$

$$r_{\min} = a - ae = a(1-e)$$

$$r_{\max} = a + ae = a(1+e)$$

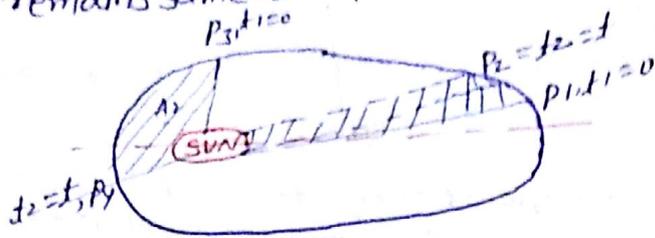
$$2a = \text{length of major axis}$$

$$2b = \text{length of minor axis}$$

$$a = \text{length of semimajor axis}$$

$e$  = eccentricity of ellipse.

B] → 2nd law: → Joining line of sun & planet sweeps equal area in a same time interval it means areal velocity (Rate of change in Area) remains same at all points of elliptical path.



$\frac{dA}{dt} = \text{Areal velo} = \text{same}$

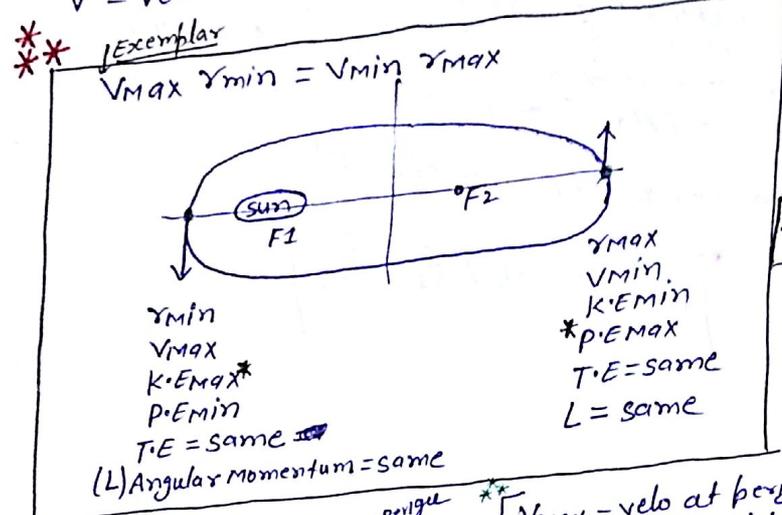
$\frac{dA}{dt} = \frac{v_r r}{2} = \frac{L}{2m} = \text{same}$

$v_r = \text{const}$

$r \uparrow \Rightarrow v \downarrow$

$r =$  Distance of planet from sun (Focus)

$v =$  velo. of planet.



$v_{max} = v_{min} \left( \frac{r_{max}}{r_{min}} \right)$

\*\*\* AIMS \*\*\*

$\frac{v_{max}}{v_{min}} = \frac{1+e}{1-e}$

→ perigee  
→ apogee

\*\*\*

$v_{max} = \text{velo at perigee point} = v_p$   
 $v_{min} = \text{velo at apogee } = v_a$

$\frac{v_p}{v_a} = \frac{1+e}{1-e} \Rightarrow e = \frac{v_p - v_a}{v_p + v_a}$

\*\*\* NOTE \*\*\*

\* If satellite/planet rotate in a circular path then orbital velo., K.E, P.E, T.E, Angular momentum remain same at all point of its path.

But in a elliptical path only T.E & Angular momentum same at all points of its path.

\*\*\* AIMS \*\*\*

# Value of  $v_{max}$  &  $v_{min}$

$v_{max} = \sqrt{\frac{GM}{a} \left( \frac{1+e}{1-e} \right)}$

$v_{min} = \sqrt{\frac{GM}{a} \left( \frac{1-e}{1+e} \right)}$

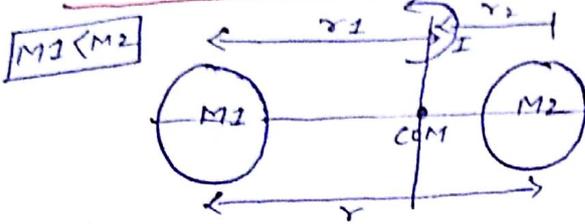
[C] → 3rd Law → Time period of planet around the sun path is proportional to  $\frac{3}{2}$  power of length of semi-major axis.

$$T \propto (a)^{3/2}$$

$$T \propto (r_{avg})^{3/2}$$

$$T_2 = T_1 \left( \frac{R_2}{R_1} \right)^{3/2}$$

# Reduced mass concept →



$$I = \left( \frac{M_1 M_2}{M_1 + M_2} \right) r^2 = \mu r^2$$

\*\* # Max & Min distance of planet from sun resp.  $r_1$  &  $r_2$  - cal distance b/w planet & sun when it is  $\perp$  to the sun →  $d = \frac{2r_1 r_2}{r_1 + r_2}$  H.M.

2005 AIEEE # Mass  $m_1$  &  $m_2$  initially  $\infty$  move in  $\oplus$ ve of gravitational attractive force cal. <sup>relative</sup> Ratio of velo. When it is 1st distance 'r' →  $V_R = \sqrt{\frac{2G}{R} (m_1 + m_2)^{1/2}}$